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RB-3610

M. A. / M. Sc. (Part - II) Examination

March / April – 2010

Mathematics : Paper - 5003

(Advanced Operations Research)

Time : **3** Hours] [Total Marks : **42**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. A. / M. Sc. (Part - 2)**

Name of the Subject :

☛ **Mathematics : P - 5003**

☛ Subject Code No. : **3 6 1 0** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) All questions are compulsory.

(4) Figures to the **right** indicate full marks.

(5) Use of commercial calculator is permitted.

1 (a) For a certain manufacturing situation, the production is instantaneous and demand is r units. Show that the optimal order quantity **6**

$$Q = \sqrt{\left(\frac{2C_3 r}{C_1}\right)\left(\frac{C_1 + C_2}{C_2}\right)}$$

where

C_1 = Storage cost per unit,

C_2 = Shortage cost per unit

C_3 = Set-up cost per run.

(b) A television manufacturing company produces its own speakers, which are used in the production of its television sets. The television sets are assembled on a continuous production line at a rate of 8000 per month. The speakers are produced in batches because they do not warrant setting up a continuous line, and relatively large quantities can be produced in a short time. **3**

Determine when and how much to produce, given the following information :

(i) Each time a batch is produced, a set-up cost of Rs. 12,000 is incurred.

(ii) The cost of keeping a speaker in stock is 0.03 per month.
(iii) Shortage of a speaker, if there exists, costs Rs. 1.10 per month.

OR

1 (a) Derive an expression for economic lot size for optimum production quantity Q per cycle of a single product so as to minimize the total average cost per unit time, when : **6**
(i) r units per time is the uniform demand rate,
(ii) production rate is k per unit time (k > r),
(iii) lead time is zero
(iv) inventory carrying cost is Rs. C_1 per quantity per unit time,
(v) shortage are not allowed, and
(vi) C_3 is the set-up cost per run.

(b) A product is produced at the rate of 50 items per day. The demand occurs at the rate of 30 items per day. Given that : set-up cost per order is Rs. 100, holding cost per unit per time is Rs. 0.05. Find the economic lot size and associated total cost. **3**

2 (a) Show that for the queuing model M/M/1:∞/FIFO, the probability that there are n units in the system is given by: **5**
$P_n = (1-\rho)\rho^n$, $n \geq 0$, ρ = traffic intensity.
Also find the average number of units into the system and the average queue length.

(b) Cars arrive at a petrol pump with exponential inter-arrival times having mean 1/2 minute. The atendant take an average 1/5 minute per car to supply petrol, the service times being exponentially distributed. Determine :(i) the average number of cars waiting to be served, and (ii) average number of cars in the queue. **4**

OR

2 (a) Obtain the steady state probabilities of M/M/C : N/FIFO queuing model. Also obtain the expressions for average queue length and average number of units in to the system. **5**

(b) A barbershop has two barbers and three chairs for customers. Assume that customers arrive in a Poisson fashion at a rate of 5 per hours, and that each barber services customers according to an exponential distribution with mean of 15 minutes. Further, if a customers arrives and there is no empty chair in the shop, he will leave. Compute : **4**
(i) What is the probability that the shop is empty ?
(ii) What is the expected number of customers in the shop?

3 (a) Define basic feasible solution for a transportation problem. Describe different methods for finding optimum solution for a transportation problem. 4

(b) Obtain optimum solution for the following transportation problem : 4

		To 1	2	3	4	Availability
From	1	8	9	6	3	170
	2	6	11	5	10	200
	3	3	8	7	9	180
Requirement		150	160	110	130	

OR

3 (a) Discuss the detail "individual replacement policy", whose maintenance costs increases with time. 4

(b) The cost of maintenance of a machine is given as a function increasing with time and its scrap value is constant. Show that the average annual cost will be minimized by replacing the machine when the average cost to date becomes equal to the current maintenance cost. 4

4 (a) Define an assignment problem. Write the algorithm to solve a given assignment problem. 4

(b) Solve the following traveling salesman problem. 4

		To 1	2	3	4	5
From	1	∞	17	16	18	14
	2	17	∞	18	15	16
	3	16	18	∞	19	17
	4	18	15	19	∞	18
	5	14	16	17	18	∞

OR

4 (a) Define the following terms : 4

(i) Pessimistic time
(ii) Critical path
(iii) Total cost
(iv) Latest start time.

(b) The following table gives the data on normal time, normal cost, crash time and crash cost for a small project : 4

Activity	*Time (days)*		*Cost (Rs.)*	
	Normal	*Crash*	*Normal*	*Crash*
1-2	3	2	300	400
2-3	3	3	30	30
2-4	7	5	420	580
2-5	9	7	720	810
3-5	5	4	250	300
4-5	0	0	0	0
5-6	6	4	320	410
6-7	4	3	400	470
6-8	13	10	780	900
7-8	10	9	1000	1200

Indirect cost is Rs. 50 per day

(i) Construct the network.

(ii) Find the critical path and normal cost and normal duration to complete the project.

(iii) Crash the relevant activity to find the optimum project time and corresponding minimum cost to complete the project.

5 (a) Write a detailed note on inventory models with m price breaks. **4**

(b) Find an optimal sequence for the following sequencing problem of 6 jobs and 4 machines when passing is not allowed of which processing time (in hours) is given below : **4**

Jobs :	1	2	3	4	5	6
Machine M_1 :	16	11	16	12	14	16
Machine M_2 :	10	7	7	6	9	11
Machine M_3 :	10	9	10	7	10	6
Machine M_4 :	12	14	12	17	16	14

Also find the total elapsed time.

OR

5 (a) Write the different characteristics of a queueing system. Also derive the distribution of arrivals. **4**

(b) Find an optimal sequence for the following sequencing problem of 5 jobs and 4 machines when passing is not allowed of which processing time (in hours) is given below : **4**

Jobs :	1	2	3	4	5
Machine M_1 :	44	46	40	47	48
Machine M_2 :	30	32	34	35	40
Machine M_3 :	39	38	35	39	35
Machine M_4 :	45	43	44	42	43

Also findthe total elapsed time.